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HW Six , Math 530, Fall 2014

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QUESTION 1. Let $(G, *)$ be a group.

(i) Let H be a subgroup of G. Let $a \in G$. Then prove that aHa^{-1} is a subgroup of G.

(ii) Assume that G has exactly one subgroup of order $n < \infty$, say H. Prove that H is a normal subgroup of G.

(iii) Assume that $|G| = nm$, where $gcd(n, m) = 1$, and suppose that G has a normal subgroup H of order m and a normal subgroup L of order n. Prove that G is group-isomorphic to $G/H \times G/L$. [Hint: Observe that $H \cap L = \{e\}$. To show that the map f is onto: note that if $a \in G$, then $a = h * l = l_1 * h_1$ for some $h, h_1 \in H$ and $l, l_1 \in L$ (Why?), and hence $a * L = h * L$ and $a * H = l_1 * H$]

(iv) Prove that $(Z_{12}, +)$ is group-isomorphic to $(Z_4, +) \times (Z_3, +)$

(v) Is $(Z_{24}, +)$ group-isomorphic to $(Z_4, +) \times (Z_6, +)$? Explain

(vi) Assume that G is cyclic of order and $|G| = n < \infty$. Prove that G is group-isomorphic to $(Z_n, +)$

(vii) Assume that G is group-isomorphic to $(Z_2, +) \times (Z_{12}, +)$. How many distinct subgroups of order 6 does G have? What about of order 2? of order 4? of order 8? explain. Prove that all subgroups of G of order 6 are isomorphic (as groups). Prove that if D_1 and D_2 are subgroups of G of order 4, then D_1 is not group-isomorphic to D_2

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